Workshop “Metodi numerici per equazioni iperboliche e cinetiche e applicazioni”

supportato da INDAM-GNCS,
responsabile progetto: **Sebastiano Boscarino**

Dipartimento di Matematica e Informatica, Catania
Sala seminari “Angelo Marcello Anile”
12 Gennaio 2018

9.45 *Opening and welcome*

10:00-10:30

Lorenzo Pareschi (Università di Ferrara)

Uncertainty quantification for kinetic equations

In this talk we will survey some recent results concerning the construction of efficient numerical methods for uncertainty quantification (UQ) in kinetic equations. In spite of the vast amount of existing research, both theoretically and numerically, the study of kinetic equations has mostly remained deterministic and ignored uncertainty. In reality, there are many sources of uncertainties that can arise in these equations:

- Incomplete knowledge of the interaction mechanism between particles/agents.
- Imprecise measurements of the initial and boundary data.
- Other sources of uncertainty like forcing and geometry, etc.

Understanding the impact of these uncertainties is critical to the simulations of the complex kinetic systems to validate the kinetic models, and will allow scientists and engineers to obtain more reliable predictions and perform better risk assessment. UQ in kinetic equations represents a computational challenge for many reasons. Simple UQ tasks such as the estimation of statistical properties of the solution typically require multiple calls to a deterministic solver. A single solver call is already very expensive for such complex mathematical models. In addition, preservation of the structural properties of the equations, like non negativity and physical conservations, is crucial for

effective performance of the numerical methods.
Recently we developed novel approaches to UQ of kinetic equations based on generalized Polynomial Chaos expansions at a particle level in order to reduce the problem dimension and maintain the main physical properties of the solution and on micro-macro Monte Carlo techniques which using control variate estimators based on the local equilibrium are capable to accelerate the slow statistical convergence of Monte Carlo methods.

References

- Carrillo J.A., Pareschi L., Zanella M., *Particle gPC methods for mean field models of swarming with uncertainties*, arXiv:1712.01677, (2017)
- Dimarco G., Pareschi L., Zanella M., *Uncertainty quantification for kinetic models in socio-economic and life sciences*, in Jin S., Pareschi L., eds, Uncertainty quantification in hyperbolic and kinetic equations. SEMA-SIMAI Series in Applied Mathematics, Springer, (2018)
- Dimarco G., Pareschi L., *Multiscale Monte Carlo methods for uncertainty quantification in kinetic equations*, work in progress
- Jin S., Lu H., Pareschi L., *Efficient Stochastic Asymptotic-Preserving IMEX Methods for Transport Equations with Diffusive Scalings and Random Inputs*, SIAM J. Sci Comp. to appear

10:30-11:00

Gabriella Puppo (Unviersità dell'Insubria)

Dissipation, dispersion and distortion of high order schemes for conservation laws

The performance of high order schemes for hyperbolic equations usually are measured through the amount of artificial dissipation and artificial dispersion with which they pollute the exact solution. These artifacts derive from the necessity to preserve stability, and from the discrete nature of the numerical solution. They are estimated applying the scheme to the linear advection equation, and they are often used to compare different schemes, of the same order of accuracy.
However, the need to prevent the onset of spurious oscillations requires that high order schemes be non linear, even when they are applied to a linear equation. This fact has been well known since the mid eighties, in the first pioneering works on high order non oscillatory schemes. What is less known, is that the non linearity of high order schemes induces spurious modes which result in the distortion of the exact solution. In this talk, I will start from the

notion of numerical diffusion and dispersion, introduce the idea of numerical distortion and propose a way of measuring it through a notion of numerical temperature. Finally, I will present high order schemes which are cool, in the sense that they are characterized by a small value of temperature, thus controlling spurious distorsive effects.

11:00-11:30

Matteo Semplice (Università di Torino)

High order Finite Volume Schemes for Balance Laws with Stiff Relaxation

The aim of this work is to construct and analyse efficient high order finite volume shock capturing schemes for the numerical solution of hyperbolic systems with stiff source terms. In standard high order finite volume schemes it is difficult to treat the average of the source implicitly, since the computation of such average couples neighboring cells, making implicit schemes extremely expensive. In this work we split the source term in a diagonal part integrated implicitly and an explicit correction term, obtaining very efficient semi-implicit schemes.

Coffee Break 11:30-11:45

11:45-12:15

Giacomo Dimarco (Università di Ferrara)

IMEX linear multistep methods for kinetic equations and related problems

In this talk, we will introduce linear multistep methods for the time integration of non linear kinetic equations. In particular, we will discuss consistency and stability of the schemes as well as their asymptotic properties in comparison with Runge-Kutta methods.

12:15-12:45

Sebastiano Boscarino (Università di Catania)

All Mach Number Second Order Semi-Implicit Scheme for the Euler Equations of Gas Dynamics

This paper presents an asymptotic preserving (AP) all Mach number _finite volume shock capturing method for the numerical solution of compressible Euler equations of gas dynamics. Both isentropic and full Euler equations are

considered. The equations are discretized on a staggered grid. This simplifes flux computation and guarantees a natural central discretization in the low Mach limit, thus dramatically reducing the excessive numerical diffusion of upwind discretizations. Furthermore, second order accuracy in space is automatically guaranteed. For the time discretization we adopt an Semi-IMplicit/EXplicit (S-IMEX) discretization getting an elliptic equation for the pressure in the isentropic case and for the energy in the full Euler equations. Such equations can be solved linearly so that we do not need any iterative solver thus reducing computational cost. Second order in time is obtained by a suitable S-IMEX strategy taken from Boscarino et al. Moreover, the CFL stability condition is independent of the Mach number and depends essentially on the fluid velocity. Numerical tests are displayed in one and two dimensions to demonstrate performances of our scheme in both compressible and incompressible regimes.

LUNCH BREAK

15:00-15:30

Roberto Ferretti (Università Roma tre)

Un solutore diffusione-trasporto esplicito, a grandi passi in tempo, per l'equazione di Navier-Stokes.

Si discuterà l'introduzione di un solutore diffusione-trasporto totalmente Semi-Lagrangiano nella soluzione della equazione di Navier-Stokes, sia nella formulazione vorticità-funzione di corrente che in quella pressione-velocita'. Nonostante il basso ordine di consistenza, questo schema si dimostra efficace e di basso costo computazionale, permettendo numeri di Courant relativamente grandi ed evitando l'introduzione di viscosità numerica indesiderata. Si presenterà lo schema, in particolare le strategie di upwinding e l'implementazione delle condizioni al bordo, e si mostreranno test numerici su benchmark classici, sia in regime laminare che turbolento.

15:30-16:00

Elisabetta Carlini (Università di Roma la Sapienza)

Una approssimazione di tipo Semi-Lagrangiano di alcune Equazioni di Fokker-Planck-Kolmogorov non lineari e applicazioni

Presento uno schema di tipo Semi-Lagrangiano per alcune equazione di Fokker-

Planck-Kolmogorov non lineari. Lo schema preserva la non negatività e la massa della soluzione e permette larghi passi in tempo. Presento un risultato di convergenza sotto ipotesi di continuità dei coefficienti e di crescenza al più lineare. Infine, mostro alcune applicazioni dello schema a vari esempi, che includono sistemi Mean Field Games e una variante del modello di Hughes per la dinamica di pedoni.

Lavoro in collaborazione con F.J. Silva

Coffee Break 16:00-16:15

16:15-16:45

Armando Coco (Università di Oxford Brookes)

A second-order finite-difference numerical method for moving interface problems in 2D and 3D

In this talk we propose a ghost-point second-order accurate numerical method to solve interface problems in 2D and 3D. The method can be applied to several problems, such as elliptic equations with discontinuous coefficients (with general non homogeneous jumps in the solution and its gradient), Euler equations of gas dynamics around moving objects, monument conservation.
The method consists of a finite-difference method on a Cartesian grid in which complex geometries (boundaries and interfaces) are embedded and tracked by a level-set method. The discretization is second order accurate in the solution and the gradient itself.
In order to avoid the drop in accuracy caused by the discontinuity of the coefficients across the interface, two numerical values are assigned on grid points that are close to the interface: a real value, that represents the numerical solution on that grid point, and a ghost value, that represents the numerical solution extrapolated from the other side of the interface, obtained by enforcing the assigned non-homogeneous jump conditions on the solution
and its flux. The linear system arising from the discretization is solved by an efficient multigrid approach. The method is robust enough to handle large jump in the coefficients: order of accuracy, monotonicity of the errors and good multigrid convergence factor are maintained by the scheme.

16:45-17:15

Chiara Stissi (Università di Catania)

On the stability of ghost point methods

The aim of this presentation is the study the stability of the ghost point methods. The first method I studied is the the Coco-Russo method for the Poisson equation. The method is the finite difference method applied to the Dirichlet problem, is an arbitrary domain.

To solve numerically such a problem we first descritize the computational domain $[0,1]^d$ on a uniform grid. To discretize the problem we proceed in two different ways. In the inside points we use the usual finite difference method. To close the linear system we must write the equations for the ghost points. For each ghost point **G** we consider its projection **F** on the border. To impose the boundary condition at **F** we condider a polynomial interpolation procedure at the node **G** and at its neighbors that depending on the accuracy we want to obtain and also on the dimension *d*.

The aim is to study the stability of the linear system which is linked to the study of the norm of the inverse matrix of the coefficients (A_h^{-1}). We have obtained numerical results in 1D and 2D and theoretical results only in 1D. The numerical experiments show that the estimates for $\left\|A_h^{-1}\right\|_p$ grow like $n^{1/p}$ p=1,2,… and the spectral radius of A_h^{-1}essentially does not depend on the position of a and it tends to a constant for large values of n. In 1D these experimental results are theoretically confirmed.

Next, I applied the ghost point method to the *gas-dynamics equations*. I started the study in one-dimensional case on a domain $[0, b] \subset [0,1]$. The general idea is that explicit differential operators in space relative to convective or material speeds are discretized by local *Lax-Friedrics* fluxes and the linear implicit operators, pertaining to acoustic waves, are discretized by central differences. I have studied the stability of the method based on the position of point b.

The aim is to study this stability on more complex geometries.