Diffusive Lotka-Volterra type systems: symmetries, exact solutions and their applications

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The main objects of this talk are diffusive Lotka- Volterra (LV) type systems, which are space versions of the classical LV dynamical system. Nowadays such systems are widely used for mathematical modeling of an enormous variety of processes in ecology, biology, medicine, physics, chemistry etc.

It is demonstrated how different types of symmetries can be used for constructing exact solutions   that satisfy typical boundary conditions and describe different scenario of population (cell, tumour, chemicals) evolution as time tends to infinity. In particular, the diffusive LV type systems used for modelling three competing species and for modelling competition between farmers and hunter-gatherers that took place thousands of years ago are studied in detail.   Several highly nontrivial exact solutions, including traveling fronts, periodic and quasi-periodic solutions and those with separation of variables are found, their properties are identified and their biological interpretation is demonstrated.

The obtained exact solutions can also be used as test problems for estimating the accuracy of approximate and numerical methods for solving boundary value problems related to the diffusive LV type systems. The talk could be interesting for specialists in Mathematical Physics, PDEs and Mathematical Modelling.

The talk is mostly based on the results obtained in collaboration with Dr. Davydovych (Institute of Mathematics of NASU, Kyiv, Ukraine) [1-2] and some recently derived results [3].

References
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 [3] P. Broadbridge, R.Cherniha, V.Davydovych, I. Marquette,   2025 (under preparation).