Pretorsion theories and their stable categories

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Torsion theories in non-additive categories have been studied by several authors in the last forty years but often in special cases concerning categories that are exact, homological, or semi-abelian, or with equalizers and pushouts, and so on. Recently, Facchini, Finocchiaro and Gran [3, 4] adopted a general approach which is based on the idea that it is possible to replace the zero object of a pointed category with a class of "trivial" objects. Thus, the notion of (pre)torsion theory can be defined in an arbitrary category $\mathcal{C}$, starting from a pair $(\mathcal{T}, \mathcal{F})$ of full replete subcategories of $\mathcal{C}$ where $\mathcal{T}$ and $\mathcal{F}$ consist of the classes of "torsion-free" and "torsion" objects, and whose intersection defines the class of "trivial objects" [3, 4]. This notion generalizes that of other torsion theories given by Bourn, Gran, Janelidze and Tholen for pointed and multi-pointed categories and under some natural mild assumptions, this new setting allows one to obtain many of the basic results that are well known for classical torsion theories in the abelian and homological frameworks.
In this talk, we describe some examples of pretorsion theories presenting the constuction of the "quotient stable category". There is a canonical quotient functor satisfiying a suitable universal property which sends the pretorsion theory into a "genuine" torsion theory. Thus this construction gives us the "universal torsion theory" or the "universal stable category" associated with a pretorsion theory.

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References

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